DISCONTINUOUS GALERKIN METHOD IN FREQUENCY-SPACE DOMAIN FOR WAVE PROPAGATION IN 2D HETEROGENEOUS POROUS MEDIA

Bastien Dupuy*, Stéphane Garambois, Jean Virieux
Laboratoire de Géophysique Interne et de Tectonophysique, Université Joseph Fourier, Grenoble, FRANCE
Tel: +33-04-76-63-52-22, E-mail: bastien.dupuy@obs.ujf-grenoble.fr
and Louis de Barros
University College Dublin, IRELAND

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ABSTRACT

Wave propagation in heterogeneous two dimensions porous media plays an important role for subsurface investigation. We consider solving dynamic wave propagation using discontinuous Galerkin method in frequency domain. This frequential approach allows us to take into account efficiently diffusive phenomena at the source and at any discontinuities including the free surface, related to Biot theory on porous medium. The propagation code is validated on simple cases with a reflectivity code which is working in wavenumber-frequency domain for stratified media. The frequency formulation better controls the energy balance at interfaces due to fluid/solid interactions. The main perspective is focused on time differential imaging by full waveform techniques in porous media.

INTRODUCTION

For many geophysical applications, related to reservoir issues (oil, gas or CO_2 storage) or to geotechnical problems, considering biphasic medium with a dynamic interaction between fluid and solid phases is essential for amplitude variations. Since pioneer works of [1] and [2] on poroelastodynamics, analytical and numerical methods have been developped to simulate wave propagation in complex media as porous media. Reflectivity methods have solved these equations in simple 3D stratified media [3] while lateral heterogeneities require volumetric methods.

The most popular of these techniques is the finite-difference method used by [4] who observe "slow" P-wave influence on attenuation in synthetic and real cases. Using the accurate spectral elements method, [5] have investigated the validity of Biot theory in media with porosity gradients: these simulations in time domain are quite cpu intensive (one has to consider memory variables for frequency-dependent dispersion mechanisms and the integration time step is quite small). Similarly, discontinuous Galerkin method (DGM) in time domain [6] successfully has computed wave propapagation in saturated anisotropic porous media with complex discontinuities.

When focusing on the full waveform inversion, one has to consider solutions in the frequency domain: therefore, we propose a 2D DGM in the frequency domain for porous media wave propagation. Indeed, this approach can consider both diffusive and propagating signals in dispersive media without any memory variables and any time integration step at the expense of solving a linear system as already shown by [7] for 2D elastic wave propagation.

First, we present how we describe porous medium as well as the Biot theory of poroelastodynamics. Then, we outline the numerical steps of the DGM constructing the linear system to be solved using a direct solver. Finally, we compare on some simple examples DGM results to semi-analytical results.

THEORY OF POROELASTODYNAMICS

[1] has determined dynamic equations which govern wave propagation in saturated porous media. Assuming a time dependency in $e^{-i\omega t}$, [8] has formulated these equations in the frequency domain giving the following system

$$\left\{ \begin{array}{l} \nabla.\boldsymbol{\sigma} = -\,\omega^2\,(\rho\,\boldsymbol{u} + \rho_f\,\boldsymbol{w}) \\ \boldsymbol{\sigma} = [K_U\,\nabla.\boldsymbol{u} + C\,\nabla.\boldsymbol{w}]\,\boldsymbol{I} + G\,[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^t - 2/3\,\nabla.\boldsymbol{u}\boldsymbol{I}] \\ -P = C\,\nabla.\boldsymbol{u} + M\,\nabla.\boldsymbol{w} \\ -\nabla P = -\,\omega^2\,(\rho_f\,\boldsymbol{u} + \theta\,\boldsymbol{w}), \end{array} \right\}$$

where the stress tensor is denoted by **σ** and the fluid pressure by **P**. The mean grain displacement is $\mathbf{u_s}$, which is approximately equal to the mean displacement of the porous medium **u**. The relative fluid-to-solid displacement **w** is given by $\mathbf{w} = \varphi * (\mathbf{u_f} - \mathbf{u_s})$. The average density and the fluid density are respectively ρ and ρ_f while mechanical properties of the porous medium defined by parameters K_u, C and M can be related to the porosity φ, the drained parameters and the mineral bulk and shear modulus (K_s, G_s) and fluid bulk modulus (K_f) through the Gassmann relations [2].

NUMERICAL METHODS

In a 2D domain, considering the solid velocity field (V_x and V_z) and the relative fluid/solid velocity field (W_x and W_z) as well as three stress components (σ_{xx}, σ_{zz}, σ_{xz}) applied to the solid and one fluid pressure P, leads to eight unknowns to be computed. As we integrate over a surface in 2D, we develop pseudo-conservative formulation. Source terms are punctual or applied stresses. Based on variational approach, a surface integration in 2D is performed inside each cell of the mesh sampling the medium. For simplicity sake, a P0 approximation, *i.e.* constant quantities inside each cell, is considered although we have also implemented higher-order lagrangian interpolation inside each cell. Using the Green's theorem, the system can be recast into a discrete vectorial form considering four flux vectors depending on stresses for velocity estimation and five vectors depending on velocities for stress estimation, giving us a discrete system. Medium parameters are supposed to be constant in each cell. Source terms (external forces and internal stresses) are the RHS expressed as a source vector **b**. Centered fluxes of stress and velocity components, explicit expressions of unknowns of the cell as well as of the three neighbouring cells, make a linear form of the LHS of the system with respect to the eight unknowns expressed as a vector **x**. We end up with a linear sparse matrix system **A . x = b** where the non-symmetrical complex impedance matrix **A** can be decomposed through a LU transformation [9], making very attractive solving the forward problem in the frequency domain when considering multi-sources as for seismic imaging.

ALGORITHM VALIDATION ON SIMPLE CASES

An explosive source is considered at the position $x = 0\ m$ and $z = 0\ m$ of a porous medium defined by parameters of a consolidated soil. The source time function is a Ricker with a central frequency of *200 Hz*. The DGM in space-frequency domain computes the stationary field for each selected frequency. For this particular case, we consider *50* frequencies between *1* and *600 Hz*. The figure 1 shows the solid vertical particle velocity for an homogeneous medium as well as the one for a higher frequency when considering two half-spaces.

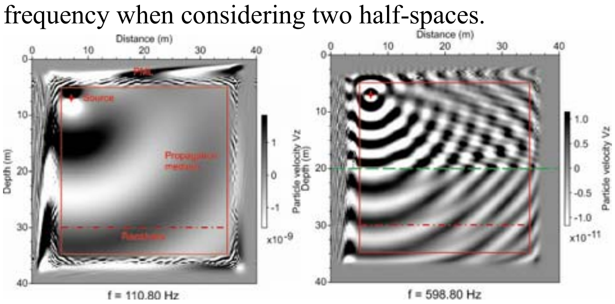

Fig. 1 Frequency maps of solid vertical particle velocity. Low frequency map in the homogeneous medium (left), and high frequency map in two-layers medium (right).

We apply an inverse Fast Fourier transform (IFFT) to each particle velocity component at selected receivers for displaying seismograms. An horizontal line of *12* receivers with a spacing of *2.5 m* is set at *23 m* deep between $x = -1\ m$ and $x = 26.5\ m$ (fig. 1). We compare DGM results with solutions from a reflectivity algorithm in 2D layered porous medium (SKB, [3]) based on the Generalized Reflection method. For an homogeneous medium, the SKB code gives the analytical exact solution. Vertical solid displacement components (u_z) between DGM and SKB codes (fig. 2) for a model with two half-spaces are in good agreement (small differences are only due to numerical errors coming from the spatial discretisation of medium in the DGM).

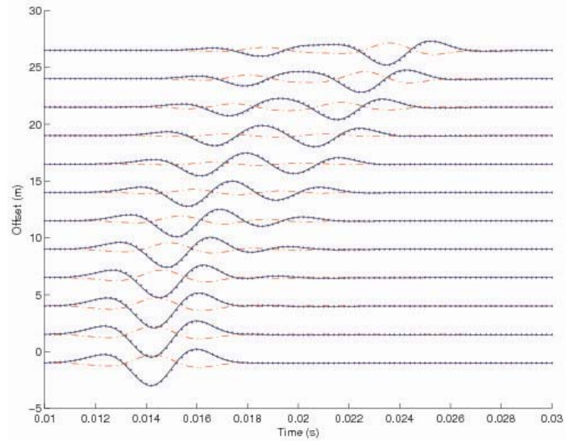

Fig. 2 Seismograms of vertical solid particle velocity in a two half-spaces medium: SKB solution in continuous line, DGM in blue crosses while red dashed lines are normalized differences between the two solutions.

CONCLUSION

We have proposed a new numerical method to simulate wave propagation in 2D heterogeneous porous media. The Discontinuous Galerkin method, in the space-frequency domain, allows us to efficiently take into account diffusive phenomena linked to Biot poroelastodynamics theory. A good agreement with reflectivity simulations in stratified media has been shown. To highlight the efficiency of space-frequency formulation, the energy loss at interfaces has to be more investigated. Then, a sensibility study of porous parameter will be carry out in order to compute sensitivity kernels. These sensitivity kernels can then be used to do time differential imaging by full waveform inversion techniques. This is a main challenge in subsurface surveys as for landslide and geotechnical issues and also in reservoir challenges to better estimate porosity, permeability and fluid properties time variations, as in CO_2 storage applications as an example.

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